Find derivatives of the following functions:

1. $f(x) = 2e^x$

Ans.

Given, $f(x) = 2e^x$

So, $f'(x) = 2\ d/dx\ (e^x)$

$= 2e^x$

2. $f(x) = 5e^x - x^2$

Ans.

Given, $f(x) = 5e^x - x^2$

So, $f'(x) = 5\ d/dx\ (e^x) - d/dx(x^2)$

$= 5e^x - 2x$

3. $f(x) = x/e^x$

Ans.

Given, $f(x) = x/e^x$

.So, $f'(x) = \dfrac{e^x\ d/dx\ (x) - x\ d/dx\ (e^x)}{(e^x)^2}$

Or, $f'(x) = \dfrac{e^x - xe^x}{(e^x)^2}$

$$\text{or, } f'(x) = \frac{e^x (1-x)}{(e^x)^2}$$

$$\text{So, } f'(x) = \frac{1-x}{e^x}$$

4. $f(x) = \frac{e^x}{(e^x+2)}$

Ans. Given, $f(x) = \frac{e^x}{(e^x+2)}$

$$\text{So, } f'(x) = \frac{(e^x+2)\, d/dx\, (e^x) - e^x\, d/dx\, [(e^x+2)]}{(e^x+2)^2}$$

$$\text{or, } f'(x) = \frac{e^x(e^x+2) - e^x(e^x)}{(e^x+2)^2}$$

$$\text{or, } f'(x) = \frac{(e^x)^2 + 2e^x - (e^x)^2}{(e^x+2)^2}$$

$$\text{or, } f'(x) = \frac{2e^x}{(e^x+2)^2}$$

5. $f(x) = e^{x2-8}$

Ans.

Given, f(x) = e^{x2-8}

So, f'(x)= e^{x2-8} d/dx $(x^2 - 8)$

Or, f'(x) = 2x (e^{x2-8})

6. f(x) =$(3 - e^{-8x})^6$

Ans.

Given, f(x) =$(3 - e^{-8x})^6$

So, f'(x) = 6 $(3 - e^{-8x})^5 \cdot \frac{d}{dx}(3 - e^{-8x})$

Or, f'(x) = $48e^{-8x}(3 - e^{-8x})^5$

7. f(s) =$(s^7+1)e-s^7$

Ans.

Given, f(s) =$(s^7+1)e-s^7$

So, f'(s) = $e-s^7 \frac{d}{dx}(s^7+1)$ + $(s^7+1)\frac{d}{dx} e-s^7$

Or,f'(s)= $7s^6(e-s^7)$ + $e-s^7(s^7+1)$ d/ds($-s^7$)

Or, f'(s)= $7s^6 e-s^7$ + $(e-s^7)(s^7+1)(-7s^6)$

Or, f'(s) = $7s^6 e-s^7$ - $7s^6 e-s^7$ - $7e-s^7$ s^{13}

Or, f'(s) = - $7e-s^7$ s^{13}

8. f(x) = $e^{-5x}+e^{2x}$

Ans.

Given, f(x) = $e^{-5x}+e^{2x}$

So, f'(x) = $\frac{d}{dx}(e^{-5x}) + \frac{d}{dx}(e^{2x})$

Or, f'(x) = -5 e^{-5x} + $2e^{2x}$

9. Find an equation of the tangent line to the graph $y=e^{-x2}$ at the point $(5, 1/e^{25})$

Ans.

Given curve $y=e^{-x2}$

So, $\frac{dy}{dx}= e^{-x2}\frac{d}{dx}(x^2)$

Or, $\frac{dy}{dx}= 2x\ e^{-x2}$

Now, $\frac{dy}{dx}$ is the slope of the tangent to the curve at any point P.

Hence at P(5, $1/e^{25}$),

$\frac{dy}{dx}= -10xe^{-25} + 51e^{-25}$

11. Determine if the function defines an inner product on R^3, where u= (u_1,u_2,u_3) and v= (v_1,v_2,v_3)

Such that $\langle u,v\rangle = 2u_1v_1+5u_2v_2+u_3v_3$

Ans.

We have,

i. $\langle u,v\rangle = 2u_1v_1+5u_2v_2+u_3v_3$

Now, $\langle v,u\rangle = 2v_1u_1 + 5v_2u_2 + v_3u_3$

$=2u_1v_1+5u_2v_2+u_3v_3$

$= \langle u,v \rangle$

ii. $\langle u,v+w \rangle = 2u_1(v_1+w_1)+ 5u_2(v_2+w_2) +u_3(v_3+w_3)$

$= 2u_1v_1+2u_1w_1+5u_2v_2+5u_2w_2+ u_3v_3+u_3w$

$=(2u_1v_1+5u_2v_2+ u_3v_3)+(2u_1w_1+5u_2w_2+u_3w_3)$

$= \langle u,v \rangle + \langle u,w \rangle$

iii. $c \langle u,v \rangle = c(2u_1v_1+5u_2v_2+u_3v_3)$

$= (2cu_1)v_1 + (5cu_2)v_2 + (cu_3)v_3$

$. \langle cu,v \rangle$

iv. $\langle v,v \rangle > 0$ and $\langle v,v \rangle = 0$

$\langle v,v \rangle = 2v_1^2 +5v_2^2+v_3^2 > 0$

And $2v_1^2 +5v_2^2+v_3^2 = 0$ occurs iff $v_1=v_2=v_3 = 0$

i.e. $v=0$

12.Determine if the function defines an inner product on R^2, where $u= (u_1,u_2)$ and $v= (v_1,v_2)$

Such that $\langle u,v \rangle = u_1v_1$

Ans.

We have,

i. $\langle u,v \rangle = u_1v_1$

$= v_1u_1 = \langle v,u \rangle$

ii. $\langle u,v+w\rangle = u_1(v_1+w_1)$

$= u_1v_1+u_1w_1$

$= (u_1v_1)+(u_1w_1)$

$= \langle u,v\rangle + \langle u,w\rangle$

iii. $c\langle u,v\rangle = c(u_1v_1)$

$= (cu_1, v_1) = \langle cu,v\rangle$

iv. $\langle v,v\rangle = v_1^2 >= 0$ and $v_1^2 = 0$ not necessarily satify V = (0,0)